

# Pressure Wave Propagation in Arteries

## *A Model with Radial Dilatation for Simulating the Behavior of a Real Artery*

**M**any distinguished scientists such as Weber, Young, and Resal have worked on pulse waves traveling in an elastic vessel [1]. The Moens-Korteweg equation is the most often cited work dealing with pressure wave propagation in an artery that has no viscoelasticity and is given by:

$$C = \sqrt{\frac{Eh}{2\rho R_i}}$$

where  $E$  = elastic modulus,  $h$  = thickness of the wall,  $\rho$  = density of the fluid,  $R_i$  = inner diameter, and  $C$  = wave velocity.

This velocity is frequency independent [1-3], and it is a well accepted index of arterial stiffness [4-6]. Womersley [7] derived the velocity and rate of flow, as well as the oscillatory motion of fluid in an artery. This work formed the foundation for most of the work on impedance in recent years [2, 7]. Both of these classical works were based on the assumption that the radial dilatation of the artery wall was small.

To solve the Navier-Stokes equation in an artery, most of the current linear theories [1-3, 7] assume the vessel dilatation to be negligible, so that the radial velocity,  $\mu$ ,  $\frac{dw}{dx}$  and  $\frac{d^2w}{dx^2}$  (where  $\mu = \frac{dr}{dt}$ ,  $w = \frac{dx}{dt}$ ), can be approximated to be zero. Womersley used this approach first and then treated the vessel dilatation as a correction [8].

More recently, to analyze flow and pressure relations, Fung [9] found that the flow depends on the fifth power of the radius in a linear expandable tube, a result which is quite good for static pressures. Stettler, et al. [10], gave an empirical relation between the cross-section of an artery and the pressure, which may be related to wave speed.

In this article, we solve the equation of pressure wave propagation in an artery with radial dilatation to give an analytic solution of wave propagation in an elastic vessel for more generalized conditions. This will serve as a complementary solution to work that is based on the assumption that the radial dilatation of the arterial wall is small. Our solution is important for physiological studies because it simulates the behavior of a real artery. To confirm our analytic solution, experiments were performed in tubes with different elastic constants.

### Theory

Up to now, most research in blood-pressure wave propagation started with the equation of motion of the fluid in the axial direction and considered the elastic wall as the boundary, so that the wall's radial motion would only contribute through the continuity equation. In this article, we apply the equation of motion in the radial direction of the elastic wall and the adherent

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fluid. After deriving pressure wave propagation in the elastic tube, the axial movement of the fluid inside the tube is then obtained through the equation of continuity.

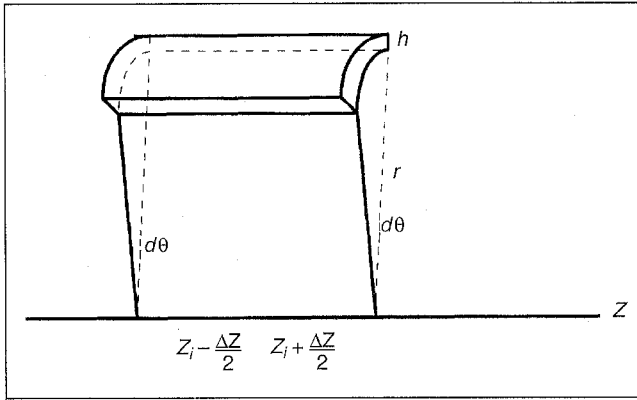
We consider a long circular cylindrical tube filled with fluid and assume that the tube material obeys Hooke's law of elasticity. The external pressure is  $P_e$  and the internal fluid pressure is  $P$ . In the static condition, the fluid pressure is  $P_0$ , the inner radius of the cylinder is  $r_0$ , and the thickness of the wall is  $h_0$ . After applying an impulse at one end of the tube, we assume that the cylinder remains circular with radius  $r$ , which is a function of  $Z$  and  $t$ ;  $Z$  being the coordinate along the cylinder.

We first derive the equation of motion of a small element that is part of the tube's  $i$ -th segment at position  $Z_i$  with axial length  $\Delta Z$ . An element with circumference angle  $d\theta$  is shown in Fig. 1.

When the radius of the segment  $r(Z_i)$  deviates from the static radius  $r_0$  by  $\delta r = r - r_0$ , the thickness,  $h$ , and the circumferential length,  $rd\theta$ , of the element changes. The existing radial strain,  $e_{rr} = \frac{\Delta h}{h_0} \approx \frac{\Delta r}{r_0}$ , and circumferential strain,  $e_{\theta\theta} = \frac{\Delta r}{r_0}$ , cause restoring normal stresses,  $E_r e_{rr}$  and  $E_\theta e_{\theta\theta}$ , by Hooke's law, where  $E_r$  and  $E_\theta$  are the Young's modulus of the wall material in the radial and circumferential directions. Thus, the total restoring force,  $F_N$ , acting on the radial plane of the small element in the radial direction becomes:

$$\begin{aligned} F_N &= -(E_r + E_\theta) \frac{\Delta r}{r_0} \Delta Z r d\theta \\ &= -E_n \frac{\Delta r}{r_0} \Delta Z r d\theta \end{aligned} \quad (1)$$

The radius of the  $(i-1)$ -th segment, immediately before the  $i$ -th segment, is  $r(Z_i - \Delta Z)$  and the radius of the  $(i+1)$ -th segment, immediately after the  $i$ -th segment, is  $r(Z_i + \Delta Z)$ . When the shearing strain,  $e_{rz} = \frac{dr}{dZ}$ , is different from zero, that is, when the small element has a bend along the axial direction, some strain energy is stored [2].



1. A small element of the arterial wall with length  $\Delta z$  and angle  $d\theta$ .

By Hooke's law, there are shearing stresses,  $\sigma_{rz} = E_{rz} \frac{dr}{dZ}$ , in the radial direction on the adjacent surfaces at  $Z_i - \frac{\Delta Z}{2}$  and  $Z_i + \frac{\Delta Z}{2}$ , where  $E_{rz}$  is the shear modulus of the tube material. Thus, the total shear force,  $F_s$ , acting on the two  $Z$ -planes, which tends to restore the column element into static curvature (parallel to the  $Z$ -axis), becomes:

$$F_s = (E_{rz} \frac{dr}{dZ} h r d\theta)_{z_i + \frac{\Delta Z}{2}} - (E_{rz} \frac{dr}{dZ} h r d\theta)_{z_i - \frac{\Delta Z}{2}} \quad (2)$$

Next, we consider the effect of the viscosity of the elastic wall and the adherent fluid by introducing a resistance force,  $F_f$ , in the radial direction.  $F_f$  is assumed to be proportional to the radial velocity  $v_r = \frac{dr}{dt}$  of the element:

$$F_f = -R_i \Delta Z r d\theta \frac{dr}{dt} \quad (3)$$

where  $R_i$  is the constant of viscosity.

From Newton's second law, the equation of motion of the element in the radial direction is then:

$$\frac{dp_r}{dt} = F_N + F_s + F_f \quad (4)$$

where  $p_r = \mu \Delta Z r d\theta v_r$  is the momentum of the element in the radial direction and  $\mu = \rho_w h + \rho_f h_f$  with  $\rho_w$  being the density of the tube material and  $\rho_f$  being the density of the adherent fluid, which moves together with the element of thickness  $h_f$ .

By substituting Eqs. (1), (2), and (3) into Eq. (4), we have:

$$d\theta \mu \Delta Z \frac{d}{dt} \left( r \frac{dr}{dt} \right) = d\theta \left\{ -\frac{E_n}{r_0} \Delta Z r \Delta r + (E_{rz} h r \frac{dr}{dZ})_{z_i + \frac{\Delta Z}{2}} - (E_{rz} h r \frac{dr}{dZ})_{z_i - \frac{\Delta Z}{2}} - R_i \Delta Z r \frac{dr}{dt} \right\} \quad (5)$$

Integrating Eq. (5) over  $\theta$  from 0 to  $2\pi$ , and noting that the

fluid volume inside the  $i$ -th segment of the tube is  $V_i = \pi r^2 \Delta Z$ , we have:

$$\begin{aligned} \mu \frac{d^2 V_i}{dt^2} + R_i \frac{dV_i}{dt} + \frac{E_n}{r_0} (V_i - V_{i0}) \\ = E_{rz} h \frac{1}{\Delta Z} \left[ \left( \frac{dV}{dZ} \right)_{z_i + \frac{\Delta Z}{2}} - \left( \frac{dV}{dZ} \right)_{z_i - \frac{\Delta Z}{2}} \right] \end{aligned} \quad (6)$$

where  $V_{i0}$  is the static volume of the fluid.

We assume that the volume compliance,  $\frac{dV_i}{dP}$ , of the  $i$ -th segment is  $C_i$ , and taking  $P_i = \bar{P}_i - \bar{P}_o$ , Eq. (6) becomes:

$$\begin{aligned} \mu C_i \frac{d^2 P_i}{dt^2} + R C_i \frac{dP_i}{dt} + \frac{E_n C_i}{r_0} P_i \\ = E_{rz} h \frac{1}{\Delta Z} \left[ \left( C \frac{dP}{dZ} \right)_{z_i + \frac{\Delta Z}{2}} - \left( C \frac{dP}{dZ} \right)_{z_i - \frac{\Delta Z}{2}} \right] \end{aligned} \quad (7)$$

For a uniform elastic vessel, let  $\Delta Z \rightarrow dZ$  and Eq. (7) can be written as:

$$\mu \frac{d^2 P}{dt^2} + R \frac{dP}{dt} + \kappa P = \tau \frac{d^2 P}{dz^2} \quad (8)$$

where  $\kappa = \frac{E_n}{r_0}$  and  $\tau = E_{rz} h$ .

If there is tension,  $T$ , in the axial direction, the term  $\tau$  may be modified by  $\tau' = E_{rz} h + T$ . In the presence of any external force, an additional force term may be added on the right side of Eq. (8).

We then define  $v_0 = \sqrt{\frac{\kappa}{\mu}} = \sqrt{\frac{E_n}{\mu r_0}}$ ,  $b = \frac{R}{\mu}$ , and

$v_\infty = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{E_{rz} h}{\mu}}$ .  $v_0$  has the dimension of frequency, and we

call it the natural angular frequency of the tube,  $v_\infty$  has the dimension of velocity, and the friction coefficient  $b$  has the dimension of 1/s.

Equation (8) then becomes:

$$\frac{d^2 P}{dt^2} + b \frac{dP}{dt} + v_0^2 P = v_\infty^2 \frac{d^2 P}{dz^2} \quad (9)$$

From this equation, we see that for a viscoelastic blood vessel, a pressure wave transmitting in the  $Z$ -direction that is mainly due to the radial oscillation of the elastic wall may be present. Assume the solution of the wave propagation equation to be:

$$P(Z, t) = P_0 e^{i(kZ - \omega t)} \quad (10)$$

where  $k(\omega) = \alpha(\omega) + i\beta(\omega)$  and

$$\alpha(\omega) = \frac{1}{v_\infty} \sqrt{(\omega^2 - v_0^2)^2 + b^2 \omega^2} \cos \frac{\theta}{2} \quad (11)$$

$$\beta(\omega) = \frac{1}{v_\infty} \sqrt{(\omega^2 - v_0^2)^2 + b^2 \omega^2} \sin \frac{\theta}{2} \quad (12)$$

with

$$\theta = \tan^{-1} \frac{b\omega}{\omega^2 - v_0^2} \quad (13)$$

then the phase velocity  $C(\omega)$  becomes:

$$C(\omega) = \frac{\omega}{\alpha(\omega)} = v_{\infty} \frac{\omega \sec(\frac{\theta}{2})}{\sqrt{(\omega^2 - v_0^2)^2 + b^2 \omega^2}} \quad (14)$$

When  $\omega = v_0$ ,  $C(v_0) = v_{\infty} (\frac{2v_0}{b})^{1/2}$  while when  $\omega \rightarrow \infty$ ,

$C(\omega) \rightarrow v_{\infty}$ . Thus, we may call  $v_{\infty}$  the high-frequency phase velocity.

The volume rate of flow,  $Q$ , of the incompressible fluid due to the fluid axial velocity can be obtained from the equation of continuity:

$$\frac{dQ}{dZ} \Delta Z = -\frac{dV}{dt} = -\frac{d(\pi r^2)}{dt} \Delta Z.$$

We define the compliance  $C_A = \frac{d(\pi r^2)}{dP}$ . By the chain rule, we have:

$$\frac{dQ}{dZ} = C_A \frac{dP}{dt} \quad (15)$$

Substituting Eq. (10) into Eq. (15), we have the volume rate of flow as:

$$\begin{aligned} Q(Z, t) &= \frac{\omega C_A}{k} P_0 e^{i(kZ - \omega t)} \\ &= \frac{\omega C_A}{\sqrt{\alpha^2 + \beta^2}} e^{-i\psi} P(Z, T) \end{aligned} \quad (16)$$

where  $\psi = \tan^{-1} \frac{\beta}{\alpha} = \frac{\theta}{2}$

This implies that flow leads pressure by angle  $\psi$ , and at the resonance frequency  $\omega = v_0$ ,  $\theta = \pi/2 \Rightarrow \psi = \pi/4$ . This result is consistent with in-vivo experimental results [11], which found that the flow leads the pressure by pulse length/8  $\Rightarrow \psi = \pi/4$  (estimated by the peaks of the simultaneous flow and pressure pulsation) at several different points and in main branches of the aorta in man.

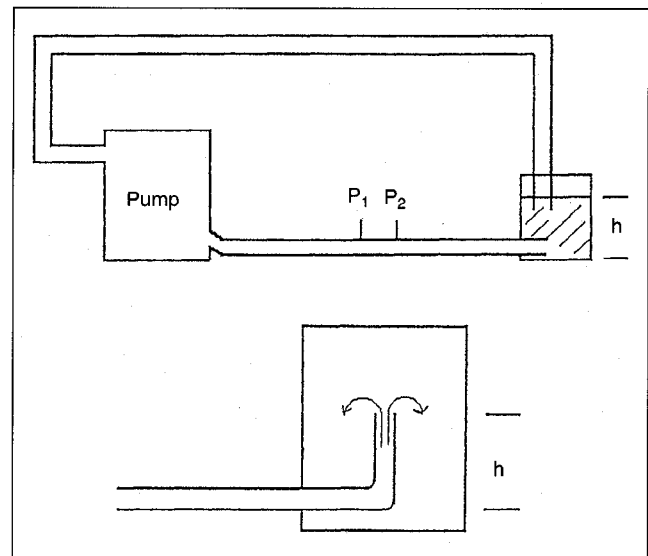
## Methods

Three tubes of the same outer diameter ( $D_0 = 1.11$  cm) and inner diameter ( $D_i = 0.95$  cm) made of latex, silicon, and tygon were first measured for  $\rho$  (density),  $E$  (force/longitudinal strain), and  $C_A$  (compliance).

To maintain static pressure, the end of the tube was bathed in a beaker with water level 10 cm or higher. We could also lift the end of the tube to let the water fall from a 10 cm height into the beaker (see Fig. 2). No difference was found in these two sets of experiments, which indicates that reflection is not important in these experiments.

Two transducers, with frequency response of 0-1000 Hz, (model DP103 Validyne Engineering Corp. Northridge, Calif.) were checked for baseline and amplification to ensure that they were equivalent. We then placed them at two different points,  $p_1$  and  $p_2$ , along the tube. The signals from the transducers were connected to a simultaneous sample and hold panel (model AX753, Axiom technology Co. LTD Taiwan, R.O.C.) and then to an A/D converter (model AX5412) personal computer interface card. The two signals were simultaneously amplified and sampled (4 kHz/per channel) and then converted to digital format.

We may check Eq. (14) by measuring pressures at  $p_1$  and  $p_2$



**2. Experimental set up of the tube experiments:** (a) The end of the tube was flat, the pressure  $P_0$  was controlled by the height  $h$  of the water level in the beaker. (b) The end of the tube was tilted, and the pressure  $P_0$  was controlled by the height  $h$  of the end of the tube.

for harmonic pulse waves of various frequency,  $\omega$ , generated by a PC-controlled stepping motor, then substitute the measured pressures into Eq. (10) to obtain:

$$\beta(\omega) = \frac{1}{\Delta x} \ln\left(\frac{A(p_1)}{A(p_2)}\right)$$

$$\alpha(\omega) = \frac{\Delta\phi}{\Delta x}$$

where  $\Delta x$  is the distance between the two points ( $p_1$  and  $p_2$ ),  $A$  is the amplitude of the pressure wave, and  $\Delta\phi$  is their phase difference.

We found pressure to be a slowly varying function of  $Z$  for most of our measurements. Thus, as a first order of approximation we may neglect  $\frac{d^2 P}{dz^2}$  and add the external impulse term  $I$ . Equation (9) then becomes:

$$\frac{d^2 P}{dt^2} + b \frac{dP}{dt} + v_0^2 P = I \quad (17)$$

From the impulse response, we found the values of  $v_0$  and  $b$  [12-14]. The measured values of  $\alpha$  of a few simple harmonic inputs (with  $\omega > v_0$ ) were then substituted into Eq. (11) to determine  $v_{\infty}$ . Both the latex and the silicon tubes were 180 cm long, with  $P_1$  at 55 cm and  $P_2$  at 65 cm for the latex tube; and  $P_1$  at 70 cm and  $P_2$  at 100 cm for the silicon tube. To check the accuracy of our theory, simple harmonic waves of  $\omega$  around  $v_0$  were generated to test if the phase velocity could be predicted by Eq. (14). For the tygon tube, we didn't try this experiment for two reasons: 1) the tygon tube was quite stiff; thus the radial dilatation was small. The impulse response also suggested that all frequencies propagated with similar resistance and that the Moens-Korteweg equation would fit the data. Moreover, since the silicon tube is stiffer than the latex tube,  $C(\omega)$  at low frequencies became somewhat larger than the theoretical values. 2) We could not generate a simple harmonic wave with frequency higher than 5 Hz with the stepping motor.

## Results

The measured values of  $\rho$ ,  $E$ , and  $C_A$  are shown in Table 1. The impulse response in the time domain is shown in Fig. 3, and Fig. 4 shows the impulse response in the frequency domain. In the time domain we did not see a second pulse traveling in the opposite direction, indicating that reflection is not important in these preparations.

The estimated values of  $b$ ,  $v_0$ , and  $v_\infty$  are shown in Table 2. We estimate the kinetic energy (KE) per unit segment of the fluid by:

$$KE \cong A\Delta z\rho v_z^2/2 = \Delta z\rho Q^2/2A$$

$Q$  is from Eq. (16). The potential energy (PE) stored per unit segment of the tube can be estimated by:

$$PE \cong \Delta z C_A P^2/2$$

and then we have  $KE/PE \cong \rho C_A c^2(\omega)/A$

From Tables 1 and 2, in cgs units, for the latex tube,  $C_A = 0.00023/980$ ,  $\rho = 1$ ,  $A \cong 1$ ,  $v_\infty = 5 \times 10^2$ , we have  $KE/PE \approx 0.06$ ; and for the silicon tube,  $C_A = 0.00013/980$ ,  $v_\infty = 1.8 \times 10^3$ , we have  $KE/PE \approx 0.43$ .

Figure 5 shows the phase velocities of the latex tube at different  $\omega$ . They fit the theoretical curve quite well, while the phase velocities of the silicon tube at different  $\omega$  in Fig. 6 follow the theoretical curve roughly. These are consistent with the basic assumption that when  $KE/PE$ , the theoretical results are good approximations.

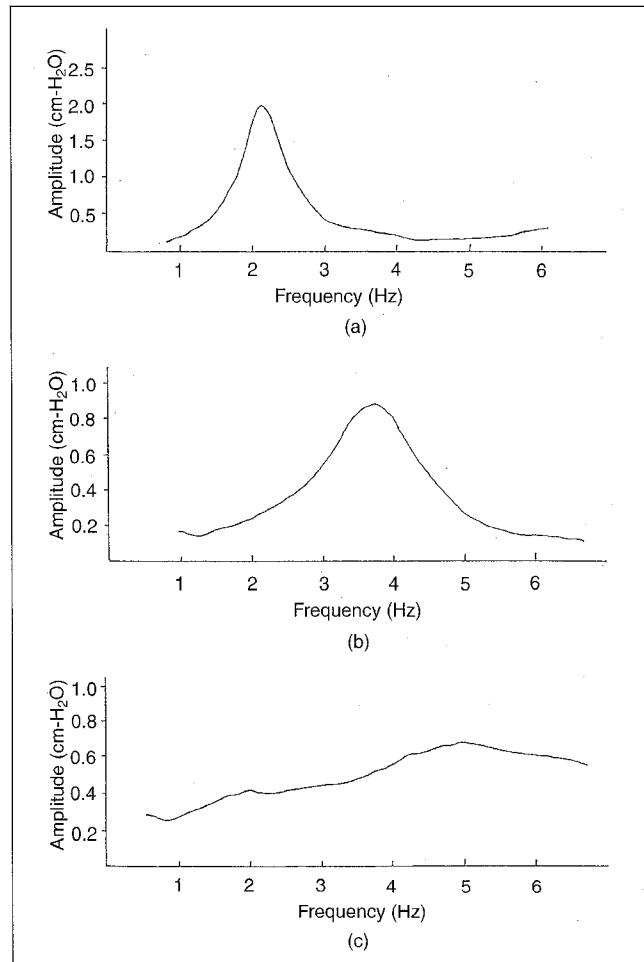
When the diastolic pressure  $\bar{P}(=\bar{P}_o - |P|)$  is not large enough, the tube might be deformed [2]; or when the elastic energy in the tube is not large as compared with the kinetic energy in the flow, the axial flow wave would interfere strongly with the radial pressure wave. Under these conditions, the equations we derived here may not hold. Fortunately, these conditions rarely happen for arteries in vivo.

## Discussion

In the derivation of the pressure wave propagation equation, we have assumed that the resistance was frequency independent and the compliance was a constant and independent of the tube length. No entrance or end effects were considered, and the energy stored in the radial dilatation was assumed to be much larger than that of the longitudinal movement of the fluid. Our solution gives a good description of wave propagation in a real tube. We also find that the softer the tube, the better the fit with theory. The latex tube has parameters  $\rho$  and  $E$  similar to a real artery in vitro, and the agreement with theory is also the best among the three tubes.

McDonald [8] found a high-frequency enhancement in the artery toward the head, and a low-frequency enhancement in the artery toward the leg. We found similar results by comparing the pulses measured on the radial artery to the pulses on the foot and on the face. The arteries on the face enrich the sixth harmonic,

tube type	$E$ $10^7$ dyne/cm <sup>2</sup>	$\rho$ g/cm <sup>3</sup>	$C_A$ cm <sup>2</sup> /cm H <sub>2</sub> O
latex	$1 \times 10^7$	0.9	0.00023
silicon	$6 \times 10^7$	1.3	0.00013
tygon	$11 \times 10^7$	1.5	0.000067



3. Impulse response in frequency domain of a: (a) latex tube at 65 cm; (b) silicon tube at 70 cm; and (c) tygon tube at 70 cm. The unit is presented in cm-H<sub>2</sub>O. However, it is the relative intensity that is important.

while the arteries on the foot enrich the second harmonic [15].

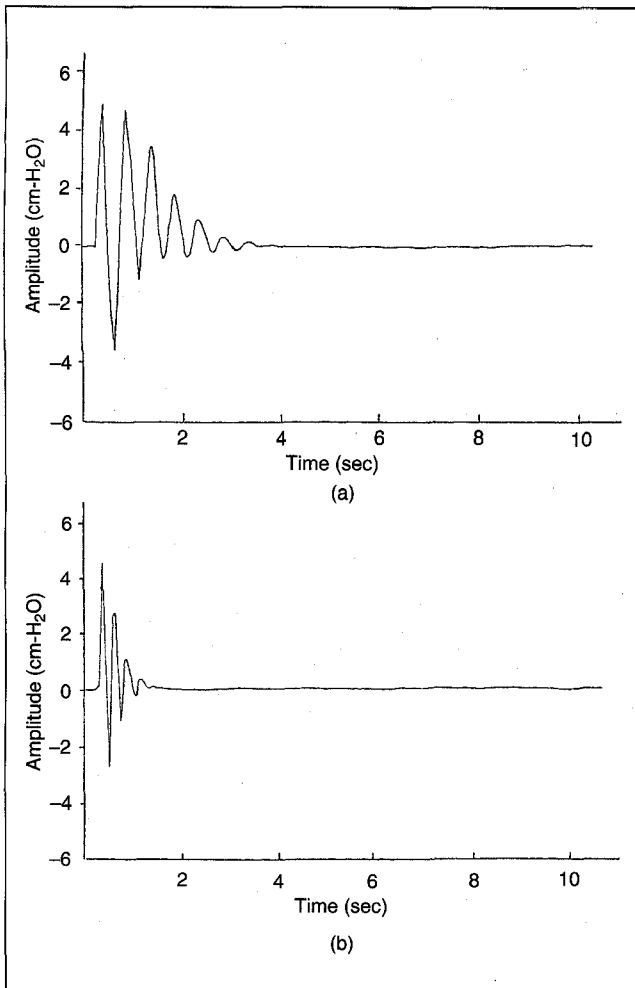
From  $v_0 = \sqrt{\frac{E_n}{\mu r_0}}$ , we see that different tubes may have different  $v_0$  if any of  $E_n$ ,  $\mu$ , or  $r_0$  are different. But the pulse wave velocity  $C(v_0) = v_\infty (\frac{2v_0}{b})^{1/2}$  can still be a good index of arterial stiffness [4-6].

If  $\rho_\omega \approx \rho_f$  and if a thick layer of solution is moving with the wall, we then have  $h_f \approx r_i$ , and the high-frequency phase velocity:

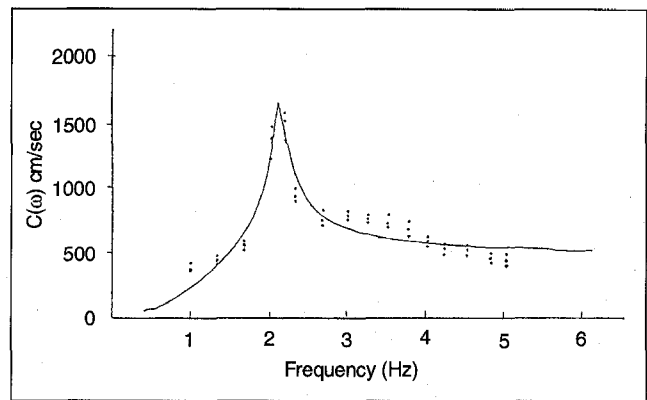
$$v_\infty \approx \sqrt{\frac{E_r h}{\rho(h+r_i)}} \approx C \text{ (Moens-Korteweg).}$$

This solution can explain many important experimental find-

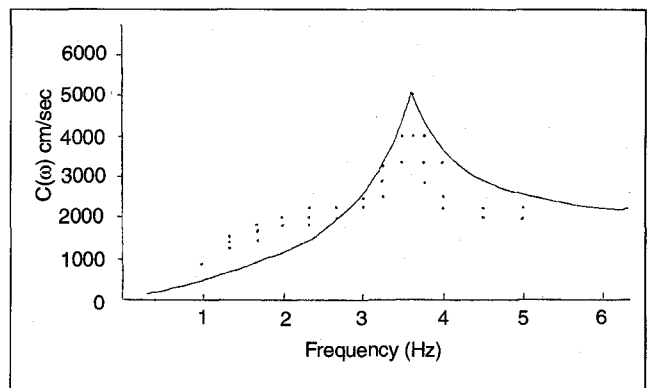
tube type	$b$ (Hz)	$v_0$ (Hz)	$v_\infty$ (cm) $\times 10^3$
latex	0.39	2.3	0.5
silicon	0.92	3.6	1.8
tygon	1.9	5.8	



4. Impulse response in time domain of *a*: (a) latex tube at 65 cm and (b) silicon tube at 70 cm. The unit is presented in cm-H<sub>2</sub>O. The *b* and *v*<sub>0</sub> in Table 2 were estimated from these data.



5. Solid line: The theoretical value by substituting *b*, *v*<sub>0</sub> and *v*<sub>∞</sub> of the latex tube in Table 2 into Eq. (14). ∴ Experimental results of *C*( $\omega$ ) by measuring  $\frac{\Delta\phi}{\Delta x}$  in the latex tube



6. Solid line: The theoretical value by substituting *b*, *V*<sub>0</sub> and *v*<sub>∞</sub> of the silicon tube in Table 2 into Eq. (14). ∴ Experimental results of *C*( $\omega$ ) by measuring  $\frac{\Delta\phi}{\Delta x}$  in the silicon tube

ings in the circulatory system; however, it does not emphasize the reflection. Taylor [16], in his reflection experiments, found that pulse-wave speed was 7400 cm/sec, which is more than 10 times the *v*<sub>∞</sub> of the latex tube and several times the *v*<sub>∞</sub> of the silicon tube, which implies that the rubber tube used in these experiments is very hard. The reflection model suggests that the amplitude of the pressure will change periodically as the wave moves along the tube, a result not observed in our experiment or other in-vivo measurements [8, 11]. Our model suggests that in a soft tube, when the reflection is not very strong, this simple wave propagation equation will fit experimental results. Due to energy consumption and blood movement, the reflection will cause energy loss and increase the load to the heart. Reflection probably increases with age, due to arteriosclerosis, and will become important in the study of pathologies [6]. In the resonance approach, frequency selection in the artery is mainly due to its elastic property. We have also shown that the arterial net in the organ and acupuncture point is equivalent to one compliance and has a specific resonance frequency [13-15, 17, 18]. This frequency selection does not completely depend on reflection—it will therefore also occur in young and healthy subjects.

This wave propagation equation together with the coupled oscillation theory complement the resonance theory in hemody-

namics [12-15, 17, 18]. It suggests that the artery and the organ select specific resonance frequencies to enhance the pressure wave. This decreases the resistance of the system and thereby facilitates blood distribution and reduces the load of the heart. A pressure wave of a specific frequency will go directly into its target organ without entering other organs [18]. This mechanism greatly improves the ability of the circulatory system to control blood distribution; it may redistribute the blood by just changing the spectrum of the pulse wave [13]. This changing spectrum mechanism can be the foundation of acupuncture as well as Chinese herb medicine [19, 20]. Physiological processes have been improved for millions of years through evolution, and they must be simple and efficient.



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